JOURNAL OF MATERIALS SCIENCE 25 (1990) 3501–3507

Some observations on liquid flow through filters

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Experiments were conducted to assess the relevance to filtration of the Poiseuille analysis of liquid flow in straight cylindrical pipes. Trials using glass filters through which water, a citric acid solution and mercury flowed showed that the analysis was qualitatively relevant. As predicted, the flow rates were a simple function of the pressure difference across the filter and were proportional to the density/viscosity ratio of the liquids. However, quantitative predictions were inaccurate because of uncertainty about filter pore dimensions and the lack of flow through narrow pores when the liquid did not wet the filter material. Flow of a suspension was further complicated by the build up on the entry faces of filters of layers of particulates which acted as secondary filters.

1. Introduction and background

The development of better quality metals and alloys depends in part on their freedom from refractory inclusions entrained during melting and casting. While foundry practices should minimize entrainment, filtration immediately prior to casting is an attractive process for restoring the quality of contaminated melts. The filters employed to remove refractory inclusions from liquid metals are themselves ceramic materials, and their efficiency can be judged in terms of the success with which they trap inclusions and on the extent to which they permit unrestricted flow of the melts. Work is in hand at this laboratory examining both of these aspects of filtration. Some experiments examining removal processes using tracer techniques to label inclusions have been reported already [1], but this paper is concerned with the flow through filters.

The flow of liquids through straight cylindrical pipes has been of interest to scientists for several centuries. The basis for our present understanding is the experimental work of Poiseuille [2] who observed that the rate of flow through a tube is inversely proportional to the fourth power of the tube radius. Subsequent analyses of these observations have been based on laminar flow concepts, so that the liquid can be imagined as a series of concentric cylinders that slide over each other as do the tubes of some telescopes. This sliding action then means that the flowing liquid experiences viscous shear. Theoretical analyses using these concepts lead to the "Poiseuille" relationship

$$Q = \frac{\pi r^4}{8\eta}\left(\frac{\Delta P}{l}\right) \qquad (1)$$

for flow through a long straight tube, where Q is the flow rate (volume/time), r is the radius, η the coefficient of viscosity, ΔP the difference between the entry and exit pressures on the liquid, and l the tube length. If the geometry of the channel changes, the term (πr^4) has to be modified but the proportionality of the flow rate to the pressure difference and the fourth power of the channel scale remains unchanged, as does the inverse proportionality to viscosity and channel length.

In practice, flow through filters is more complicated than the phenomena described by Poiseuille. While flow through a filter has been shown by Darcy [3] and subsequent workers to be dependent on the pressure gradient, Kozeny [4], Carmen [5] and others have shown it also to be influenced by the relative porosity and surface area of filters. Further the liquid has first to penetrate the filter before it can pass through, and hence capillary attraction or repulsion effects may be of significance. Similarly, the flow paths through a filter may be tortuous and variable so that the channel scale term may be difficult to predict. Further, flow may be rendered more difficult as particles removed from the liquid accumulate at the entry face or within the filter. Some of these effects were examined in this work through model experiments conducted at room temperatures. Particular attention was paid to the applicability of the Poiseuille relationship and to the influence of liquid-filter wettability on the initiation and rapidity of flow.

2. Experimental materials and methods

Sintered borosilicate glass filters were used in this study with nominal pore sizes of 200 ± 50 μm, 120 ± 30 μm, 65 ± 25 μm, 27 ± 13 μm and 10 ± 5 μm. They were 5 mm thick and 10 mm diameter and were sealed into the waisted mid-sections of 13 mm i.d., 700 mm long glass tubes. Before each experiment, the filter tube was ultrasonically degreased in acetone for 10 min and dried with a hot-air blast.

Rates of flow were measured at room temperature,

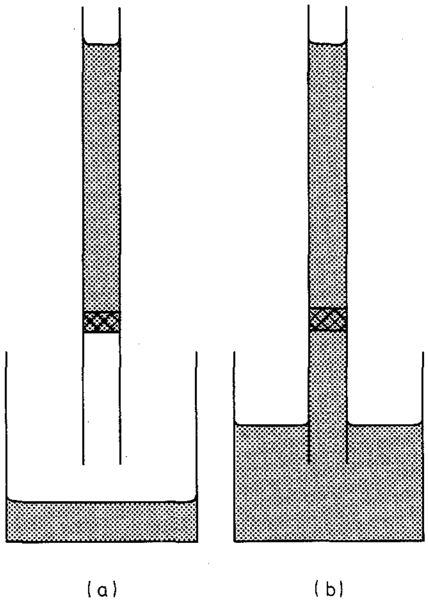

Figure 1 Experimental arrangements.

18 to 20.5° C, by suspending the filter tube above a collection bath as shown in Fig. 1a, filling the upper entry tube to a depth of 100 to 500 mm and timing the rate of decrease of the column height, h. For this arrangement, Equation 1 can be rewritten as

$$\frac{d(\pi R^2 h)}{dt} = \frac{(N\pi r^4)}{8\eta}\left(\frac{h\rho g}{l}\right)$$

and as

$$\left(\frac{dh}{h}\right)\frac{1}{dt} = \frac{Nr^4 \rho g}{8R^2 \eta l}$$

which can be integrated to yield

$$\ln h = \frac{Nr^4 \rho g}{8R^2 \eta l} t + \ln h_0 \qquad (2)$$

where R is the radius of the tube, N the number of channels in the filter, ϱ the liquid density, t the time and h_0 the column height at time $t = 0$.

Experiments were conducted with a variety of fluids. Particle-free liquids (water, mercury, water–Teepol solutions, and a citric acid solution) were used to assess the relevance of the Poiseuille relationship to flow through a filter and the influence of wettability. Additionally, experiments were performed with a suspension of coarse, 500 to 10 μm, perspex particles in 10% aqueous citric acid. When necessary, experimental work other than flow trials were performed and reference will be made to porosimetry, viscometry and to optical microscopy.

TABLE I Filter characteristics

Filter type	Nominal pore radius (μm)	Mean pore radius (μm)	Open porosity (%)	N†
1	75–125	> 40	6.3	
2	45–75	40	24.6	3.8×10^3
3	20–45	19	25.2	1.7×10^4
4	7.5–20	11.5	37.7	7.1×10^4
5	2.5–7.5	4.5	97.9	1.2×10^6

*Derived from mercury porosimetry measurements.
† Estimated number of open pores in the filter.

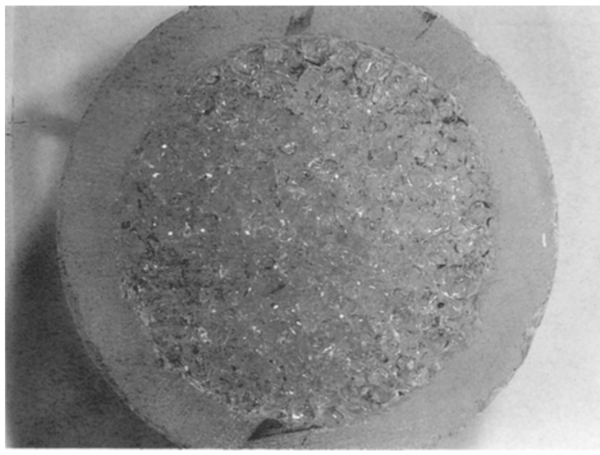

Figure 2 Micrograph of a Type 1 filter, × 4.7.

3. Results

3.1. Filter structure

The filters were close packed semi-sintered bodies of silica microspheres as illustrated in Fig. 2. Identifying individual pathways by cross-sectioning was impractical, but distributions of pore sizes were obtained for the various filter grades using a Carlo Erba mercury porosimeter. As can be seen in Fig. 3, the pore sizes present in an individual filter could vary considerably. (In this and in subsequent figures the curve labels identify the filter types, Table I). The mean pore sizes were in reasonable accord with the manufacturers "pore size" for the finer filters, but not for the coarser filters, because these contained many pores with diameters greater than 100 μm and hence were above the upper limit of measurement for the porosimetry equipment. Where necessary later, the measured mean values will be used except for the coarsest filter. Also included in the table are the percentages of the filter volumes that were open porosity.

3.2. Water flow

Two types of experiment were conducted, the first with the arrangement shown in Fig. 1a. An initial water height of 450 mm was used and as shown in Fig. 4 the observed rates were slower for the finer

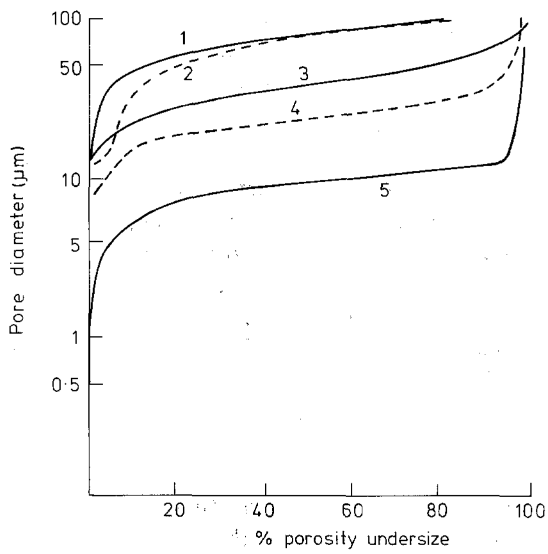

Figure 3 Mercury porosimetry data.

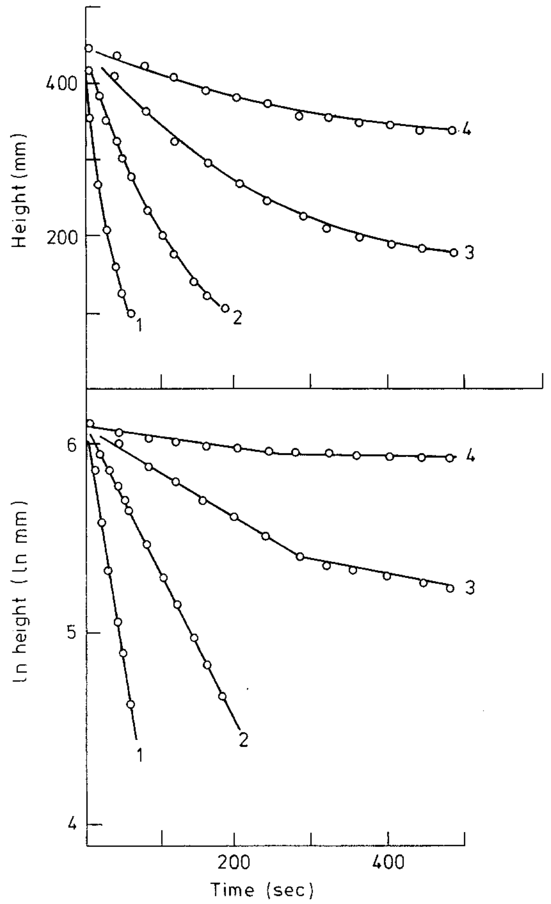

Figure 4 Flow-rate data for water using the arrangement shown in Fig. 1a. The curve labelling indicates the filter type.

TABLE II Flow rate data

Fluid	Filter	Rate* (sec^{-1})	
Water			
	1	0.0240	
	2	0.0080	
	3	0.0024†	(0.0008‡)
	4	0.0005†	(0.0002‡)
Water, both faces covered			
	1	0.0240	
	2	0.0066	
	3	0.0026	
	4	0.0006	
Water plus detergent			
0% detergent	3	0.0025	
1	3	0.0016	
2	3	0.0012	
3	3	0.0009	
Mercury			
	2	0.0444	
	3	0.0514	
	4	0.0004	
Citric acid			
	1	0.0215	
	2	0.0089	
	3	0.0023	
	4	0.0007	
Perspex suspension			
	1	0.0089†	0.0066‡
	2	0.0035†	0.0022‡
	3	0.0013†	0.0007‡
	4	0.0008†	0.0002‡

*Derived from logarithmic plots.
†Initial rate.
‡Final rate.

filters and, further, decreased progressively as the water flowed through them. To test the relevance of these data to the Poiseuille relationship, the column height data were replotted as logarithms and linear rates were revealed, Fig. 4b, in accordance with Equation 2. However, flow through the finer filters generated two logarithmic linear relations, with the final slopes being significantly smaller, Table II.

Direct observations of the filter exit faces revealed phenomological changes in flow behaviour that could be associated with the rate changes. When the flow rates were high, water passed through the filters to exit by cascading down the tube walls and by raining as droplets from the filter face. With the finer filters, droplet formation ceased after a few minutes when there was a change in the slopes of the logarithmic height plots. Later, the centres of some of these filters dewetted and hence a proportion of the channels ceased to be as effective because their existing water could be removed only slowly by traversing over the dried-out surface.

To avoid the influence of removal processes, a second series of experiments used the arrangement shown in Fig. 1b in which the tube below the filter was occupied by a water column. Hence the exciting water merely mixed with that already in contact with the filter. Prior to this series of experiments, the filter tubes were completely immersed in the water bath for 60 min, and care was taken to ensure that the upper and lower water columns remained intact when the tubes were raised into the arrangement shown in Fig. 1b. Once more, water flow rates were measured by timing the decrease in level of the surface of the upper column. An initial height of 400 mm was used and as shown in Fig. 5 flow was slower through the finer filters and decreased progressively with time. Replotting the height data as logarithms revealed single linear relationships for flow through all the filters, the slopes of which closely resembled those for the initial flows in the first series of experiments, Table II. In order to avoid complications in interpretation, therefore, this configuration was adopted as the standard.

3.3. Effects of varying wettability

Water wetted the borosilicate glass of the filters with a contact angle of 30° as measured by sessile drop tests, and the influence of wettability on flow behaviour was assessed by experiments using water whose behaviour was modified by the addition of a detergent and by experiments with mercury, which did not wet the glass.

The addition of Teepol detergent caused the flow rates of water to decrease as illustrated by the single sloped logarithmic plots in Fig. 6 for a Type 3 filter. Other liquid characteristics changed by the addition of Teepol were the density, albeit by only 0.1%, its ability to wet, and its viscosity. The addition of only 1 vol % Teepol decreased the contact angle of the liquid to 0° so that it was able to spread indefinitely over the

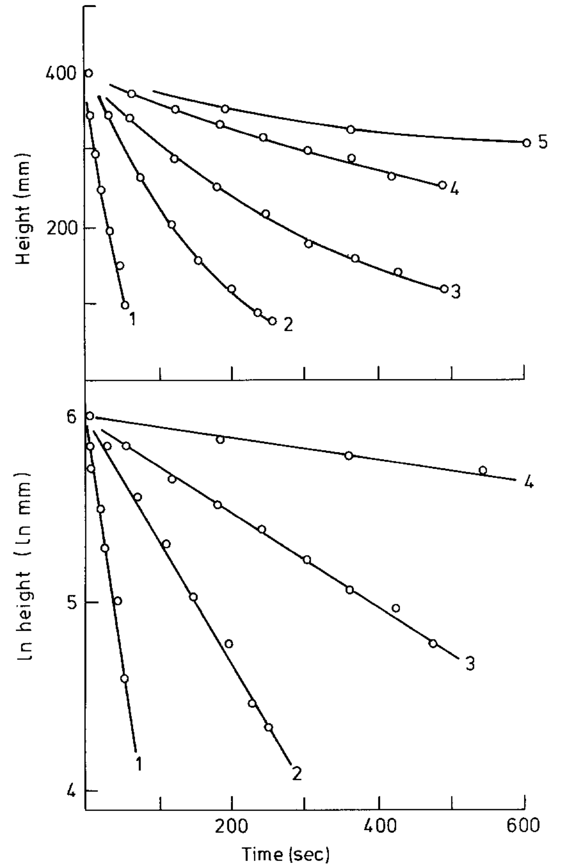

Figure 5 Flow-rate data for water using the arrangement shown in Fig. 1b.

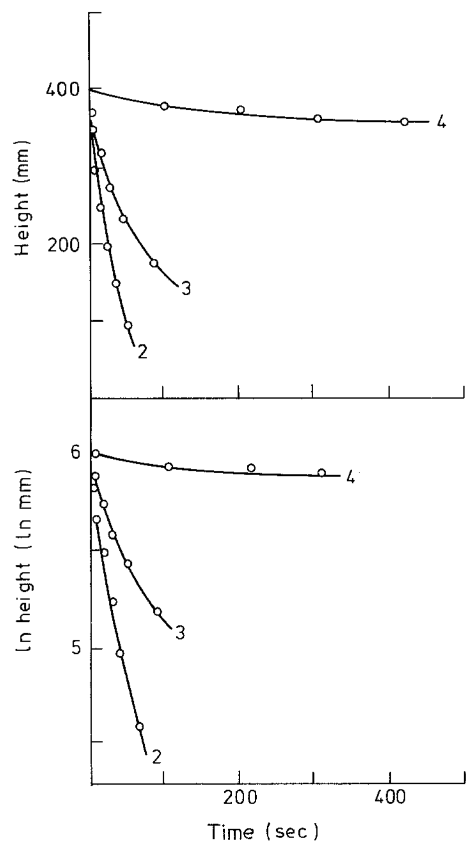

Figure 7 Flow-rate data for mercury.

glass surface. However, the effect of Teepol additions on viscosity, as measured by a Brookfield equipment, was progressive increasing from 1.00 to 1.68, 1.91 and 2.71 mN sec m^{-2} when 1, 2 and 3 vol % were added, respectively. In fact there was a simple inverse relationship between the logarithmic flow rates and the viscosity in accord with Equation 2; values of the product $\eta(\mathrm{d}\ \ln h/\mathrm{d}t)$ range from 2.3 to 2.7 μN m^{-2}.

The behaviour of mercury was more complex, with no flow occurring until well-defined liquid heights were exceeded; 218 mm for a Type 4 filter, 168 mm for a Type 3 and 92 mm for a Type 2. Once flow was initiated it continued even after the liquid level fell below the initiating height. However, flow did cease before all the mercury had passed through the filters, to leave residual heights of 152 mm for a Type 4 filter, 109 mm for a Type 3 and 56 mm for a Type 2. When the mercury flowed, it did so more rapidly through the coarser filters but at rates that decreased progressively with time, Fig. 7. This behaviour superficially resembled that displayed by wetting water, but linear time dependencies were not revealed when height logarithms were plotted. To obtain linear relationships it was necessary to plot the logarithm of the height above the level at which flow ceased, Fig. 8.

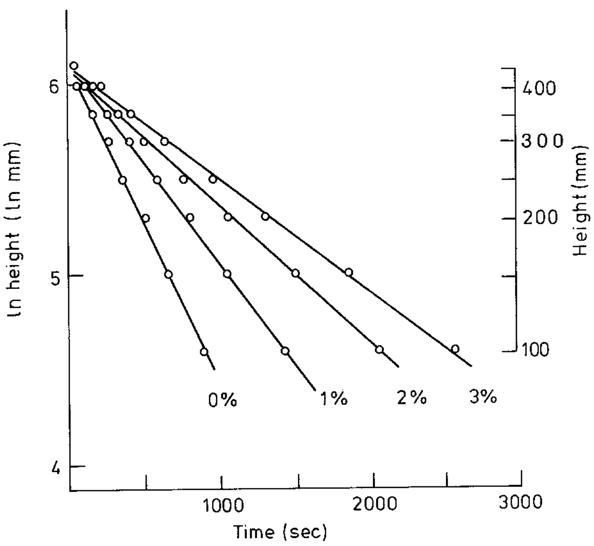

Figure 6 Flow-rate data for water–Teepol solutions using a Type 3 filter.

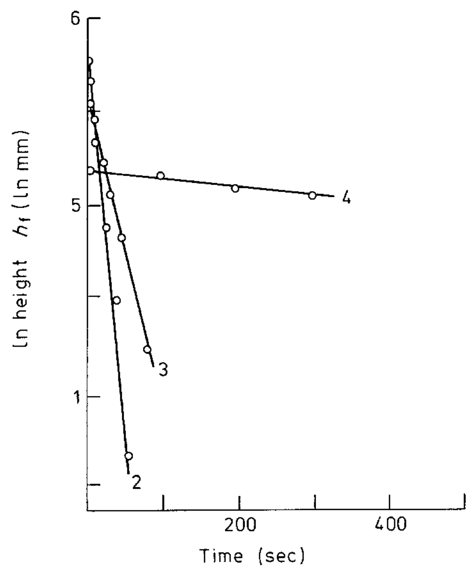

Figure 8 Replotted flow-rate data for mercury taking account of the residual height, h_f.

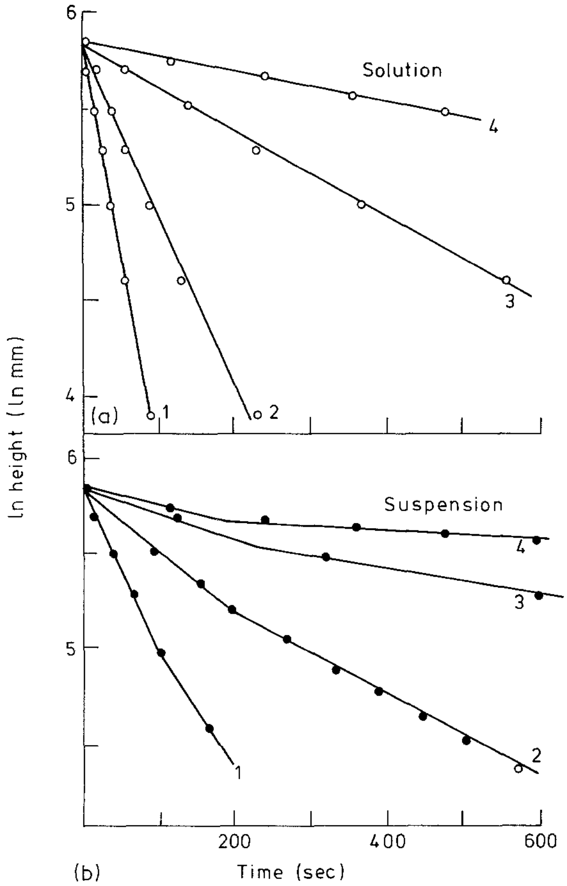

Figure 9 Flow-rate data for, (a) a 10% citric acid aqueous solution and, (b) a citric acid-perspex suspension.

3.4. Suspension flow

Before using the citric acid solution–perspex suspension, the behaviour of the particle-free 10% citric acid aqueous solution was characterized. This closely resembles that of water, as illustrated by logarithmic data plots in Fig. 9, and the slopes quoted in Table II.

The suspension with perspex, however, flowed much more slowly except through the Type 4 filter, and the plots of the height logarithms could not be represented by single linear relationships, Fig. 9. The initial slopes of these suspensions were about half of those for the citric acid solution except for flow through the Type 4 filter, while the final slopes were about a quarter of those for the citric acid solution except for flow through the coarse Type 1 filter, Table II. During the flow, cakes of perspex particles built up on the entry faces of the filters. The material deposited on the filter surface was finer than that trapped further out, Fig. 10. For a Type 3 filter the diameters of the finer trapped material were about a tenth of a millimetre while those of the outer layer were about half a millimetre. The cakes were typically about a millimetre thick and the region of fine particles was about two layers deep.

4. Discussion

The primary objective of this work was to assess the relevance of the Poiseuille relationship to flow through filters, and the principal conclusion that can be reached is that the relationship is indeed relevant. Poiseuille's studies with straight cylindrical tubes showed that the rates of flow increased rapidly with the tube radii, obeying a fourth power relationship, and with the pressure differences along the lengths of the tubes. The channels in our filters were not cylindrical or straight, but nevertheless flow rates increased rapidly with the coarseness of filters. Further, in this study the pressure difference inducing flow was due to the height of the liquid column above the filter and in every case the rate of flow decreased as the column heights decreased. Qualitatively, therefore, the Poiseuille relationship can be used to describe aspects of flow through filters. Quantitative descriptions, however, are not as satisfactory.

The slopes of the plots of height logarithms against time, Equation 2, depend on the liquid parameters ϱ and η, the filter tube radius R, the filter characteristics N, r and l. Values of ϱ and η are cited in reference books ([6] pp. 71, 183) and R is readily measurable.

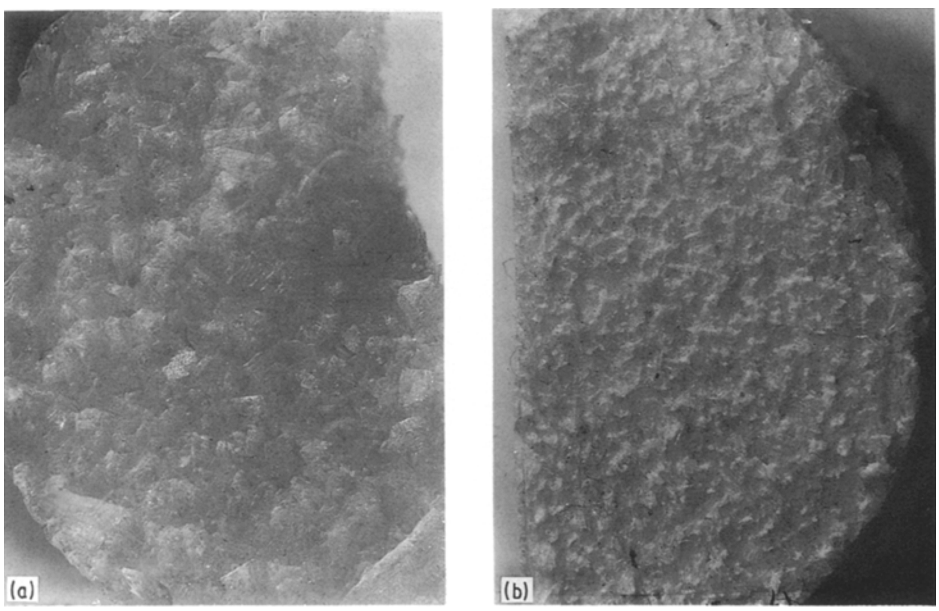

Figure 10 Micrographs of perspex particles deposited on the entry face of a Type 3 filter, × 10.8. (a) Outer surface, (b) inner surface adjacent to the filter.

Mean values of r derived from mercury porosimetry are summarized in Table I, and values of N can be derived from the filter porosity and mean pore radii by substitution in the relationship

$$N = \frac{\pi R_F^2}{\pi r_M^2} P_F \quad (3)$$

where R_F is the filter radius, r_M the mean pore radius, and P_F the fractional porosity of the filter. As can be seen from Table I, the N values vary by two orders of magnitude from the finest to the coarsest filters.

Substituting values of $10^3\,\mathrm{kg\,m^{-3}}$ and $10^{-3}\,\mathrm{N\,sec\,m^{-2}}$ for the density and viscosity of water yields predicted slopes of 0.0571 to 0.002 87 for the Type 2 to Type 5 filters, as shown in Table III, when r is equated the mean radii derived from mercury porosimetry, and the pore length l is equated to the filter thickness. These slopes are about five to ten times greater than those observed in practice, which implies that the values substituted have under-estimated l or over-estimated N or r. Because the paths through a filter are tortuous, the substituted value for l is clearly an under-estimate. Further the mean pore radius is probably an over-estimate of r because a constriction at any point along the path would decrease the flow rate. When these uncertainties are taken into account, an order of magnitude agreement between theory and experiment is not too unreasonable.

If the liquid is changed, the flow rates should vary as does the ratio ϱ/η and this is observed in practice. Thus the flow behaviour of the aqueous 10% citric acid solution was very similar to that of pure water as might be expected, while the addition of 3 vol % Teepol decreased the flow rate of water by 64% in excellent accord with 63.1% decrease in the ϱ/η ratio. The ρ/η ratio for mercury as for most molten metals is high ([6], p. 71) 8.8 times that of water, and the observed flow rates through the Type 2 and Type 3 filters were faster for mercury than water by 6.7 times and 5.9 times. The rate through the fine Type 4 filter was less for mercury than water, Table II, but this anomalous behaviour is believed to be due to another factor – wettability.

Water wets the borosilicate glass of the filters and flow should occur through every open pore. Mercury, however, does not wet glass and will experience a capillary repulsion that must be overcome if it is to enter the filter and flow through it. This requires a force to be exerted and accounts for the lack of flow until certain upper column heights were exceeded. This height depends on the radius of the pore. If the

TABLE III Calculated flow rates*

Fluid	Filter type	Rate ($\mathrm{sec^{-1}}$)
Water	2	0.057 10
	3	0.013 19
	4	0.007 23
	5	0.002 87
Mercury	1	0.500 60
	2	0.115 98
	3	0.063 31
	4	0.025 20

*Equal to $Nr^4\varrho g/8\eta l R^2$ in Equation 2.

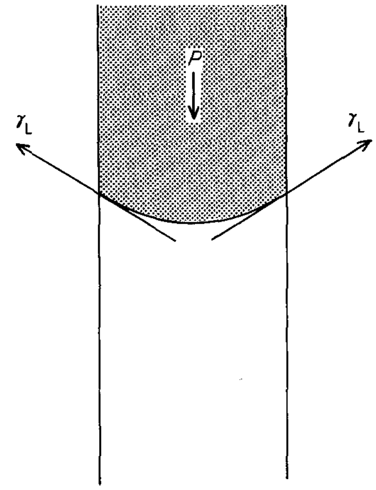

Figure 11 Forces acting on a non-wetting liquid front in a vertical capillary.

liquid front shown in Fig. 11 advances a small distance, δL, the work done will be δW, where

$$\delta W = P\pi r^2\,\delta L + (\gamma_S - \gamma_{SL})\,2\pi r\delta L \quad (4)$$

and γ_S and γ_{SL} are the energies of the tube surface and of the tube–liquid interface. From the Young equation [7]

$$\gamma_S - \gamma_{SL} = \gamma_L \cos\theta$$

where γ_L is the energy of the liquid surface and θ is the contact angle assumed by the liquid front. Substituting into Equation 4 and making δL zero to define the equilibrium condition, the relationship

$$h\varrho g r = -2\gamma_L \cos\theta \quad (5)$$

is obtained. For mercury, γ_L is $0.6\,\mathrm{J\,m^{-2}}$ ([6] p. 111), and while θ for the particular filter glass is not known, that for silica is 125° [8]. Assuming all the pores have the mean radius, the initiating column height above the filter entry face should be 272 mm for a Type 3 filter. In practice it is only 168 mm. Agreement between prediction and practice is somewhat better for the coarser filters, but some discrepancy can be expected because the filters contain a range of pore sizes.

The non-wetting behaviour of mercury may be partially responsible for the observed flow rates being lower than those predicted. The filters contain a range of pore sizes and hence the finer pores may not have been penetrated, decreasing the effective value of N in Equation 2, and as might be expected this effect is particularly significant for the finer Type 4 filter.

Pressure is needed to maintain as well as to initiate the flow, and flow ceased when the upper column heights fell below critical heights of 152 to 56 mm. The ratios of these terminating heights to those needed to initiate flow fall between 0.6 and 0.7 for the filters used. Now the contact angle of 125° referred to earlier was for a liquid front advancing over a solid surface, but for many systems there exists a somewhat smaller angle characteristic of liquid fronts receding from solid surfaces. That is, while a certain pressure has to be exerted initially to overcome capillary repulsion and allow flow to start, flow will cease and the liquid will retract from the channels if the pressure is allowed to fall below that determined by the smaller receding front contact angle, and the height ratio of 0.6 to 0.7 suggests contact angle values of 110 to 114° for the receding fronts.

The behaviour of particle-free liquids can be complex, but some quantitative analysis can be attempted. Our study of the behaviour of a suspension, however, can be discussed only in qualitative terms. While linear time dependencies can be observed in the plots of height logarithms against time in accord with the Poiseuille relationship, the data plots had breaks to yield an initial and a final flow rate, neither of which could be compared with theoretical predictions because of the lack of viscosity and density data. In considering the behaviour of the perspex–citric acid mixture, breaks in flow rates shown in Fig. 11 can be expected due to the formation of cakes on the entry face of the filters. However, it is surprising that the logarithmic height data assumed a second linear time dependence because the thickness and structure of the cakes must change progressively as filtration proceeds. In principle, it should be possible to calculate the pore size within the finest region of the cake from the final suspension flow rates if it is assumed that paths in the cake and the filter are equally tortuous. It is notable that these final rates are about one quarter of those for the citric acid solution, Table II, and if l for the cake is taken as 0.2 mm, as compared 5 mm for the filter, the radii ratio is about three. Whether finer perspex particles were trapped first or penetrated the coarse cake and were then trapped within it is not clear and is difficult to assess unless some way is developed to label the particles.

5. Conclusions

1. Experiments in which a column of a liquid or a suspension flows through a glass filter can be used to assess the relevance of the Poiseuille relationship to filtration. The flows of liquids in these experiments were in qualitative accord with the Poiseuille relationship. The logarithms of the column heights decreased linearly with time and the actual flow rates reflect the density/viscosity ratios of the liquids.

2. The actual rates of liquid flow cannot be predicted accurately because of lack of detailed information about path lengths and pore sizes.

3. Removal of liquid leaving filters can affect flow behaviour as can the inability of the liquid to wet the filler.

4. The behaviour of a suspension is complex because filter cakes can form that throttle flow of the liquid.

Acknowledgements

This work was conducted with the financial support of the Metals Committee of the UK Department of Trade and Industry.

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Received 24 February
and accepted 30 August 1989